Conduction Z-transfer Function Coefficients for Common Composite Wall Assemblies

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ABSTRACT

Results of the conduction z-transfer function coefficient calculations are presented for clear walls and separated details listed in the ASHRAE research project, 1145-TRP, "Modeling Two and Three-dimensional Heat Transfer through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs."

Resistances, three-dimensional response factors, and the so-called structure factors have been computed using the finite-difference computer code HEATING 7.2. The z-transfer function coefficients were derived from the set of linear equations (relationships with the response factors), which are to be solved using minimization procedures.

Comparison of the response factors and z-transfer function coefficients for three-dimensional and one-dimensional equivalent wall models indicates that, in general, the one-dimensional model produces some delay in dynamic thermal response to the external excitation, whereas response to internal excitation may be overestimated.

Test simulations show perfect compatibility of the heat flux calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients derived from the response factors.

INTRODUCTION

The method of derivation of the conduction z-transfer function coefficients from the response factors for three-dimensional wall assemblies is presented.

Response factors, which represent surface heat flow due to triangular temperature excitations at discrete time instants, are calculated with the help of a computer code to simulate three-dimensional heat conduction. They are used as the "input data" to determine z-transfer function coefficients from the set of linear equations, which includes relationships with the response factors and compatibility conditions. This infinite set of equations is to be solved applying cut off and using minimization procedures.

RELATIONSHIPS BETWEEN RESPONSE FACTORS AND Z-TRANSFER FUNCTION COEFFICIENTS

In terms of the response factors, heat flux across the interior surface of a wall element at time instant $n\delta$, $Q_{i,n\delta}$ can be represented as follows (Kusuda 1969; Clarke 1985):

$$Q_{i,\,n\delta} = \sum_{k=0}^{n} [X_n T_{i,\,(n-k)\delta} - Y_n T_{e,\,(n-k)\delta}] \tag{1}$$

where $\{T_{i,n\delta}\}$ and $\{T_{e,n\delta}\}$ are sequences of the ambient (or surface) temperatures values, and $\{X_n\}$ and $\{Y_n\}$ are sequences of the response factors.

As far as three-dimensional problems are concerned, heat flux values in Equation 1, as well as response factors, are to be understood as averages over the surfaces of a wall element, adiabatically cut from rest of the wall. Driving temperatures

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are functions of time only and do not depend on spatial coordinates, which is also the case when boundary conditions of the first kind are assumed. Dimensions of the element and location of the cut surface are to be established when creating its three-dimensional model in order to determine its thermal characteristics.

The z-transform of the interior heat flux, $Z[Q_i]$ is related to the z-transforms of the interior and exterior temperature, $Z[T_i]$ and $Z[T_e]$, by the following equation (see Jury 1964):

$$Z[Q_i] = Z[\{X_n\}]Z[T_i] - Z[\{Y_n\}]Z[T_e] \tag{2}$$

where $Z[\{X_n\}]$ and $Z[\{Y_n\}]$ are the z-transforms of the sequences of the response factors X_n and Y_n.

$$Z[Q_i] = \sum_{n=0}^{\infty} Q_{i,n\delta} z^{-n},\ Z[\{X_n\}] = \sum_{n=0}^{\infty} X_n z^{-n},\quad Z[\{Y_n\}] = \sum_{n=0}^{\infty} Y_n z^{-n} \tag{3}$$

The condition response factors X_n and Y_n should satisfy

$$\sum_{n=0}^{\infty} X_n = \sum_{n=0}^{\infty} Y_n = \frac{1}{R}, \tag{4}$$

which is equivalent to the following condition for the z-transforms $Z[\{X_n\}]$ and $Z[\{Y_n\}]$:

$$\lim_{z \to 1} Z[\{X_n\}] = \lim_{z \to 1} Z[\{Y_n\}] = \frac{1}{R}. \tag{5}$$

R denotes the resistance per unit surface area, determined from the average heat flux in steady-state conditions.

Now let $Z[\{X_n\}]$ and $Z[\{Y_n\}]$ be given as the quotients:

$$Z[\{X_n\}] = \frac{1}{R}\frac{C(z)}{D(z)},\ Z[\{Y_n\}] = \frac{1}{R}\frac{B(z)}{D(z)} \tag{6}$$

where

$$B(z) = \sum_{n=0}^{\infty} b_n z^{-n},\ C(z) = \sum_{n=0}^{\infty} c_n z^{-n},\quad D(z) = \sum_{n=0}^{\infty} d_n z^{-n}. \tag{7}$$

The dimensionless conduction z-transfer function coefficients are b_n and c_n, which correspond to the coefficients b_n, c_n from the *ASHRAE Handbook—Fundamentals* (1989, 1997), multiplied by R.

Equation 2 can be rewritten in the form:

$$D(z)Z[Q_i] = \frac{1}{R}\{C(z)Z[T_i] - B(z)Z[T_e]\}. \tag{8}$$

Equation 1, for $Q_{i,n\delta}$, assuming $d_0 = 1$, is now replaced by (Stephenson and Mitalas 1971)

$$Q_{i,n\delta} = \frac{1}{R}\left[\sum_{m=0}^{n} c_m T_{i,(n-m)\delta} - \sum_{m=0}^{n} b_m T_{e,(n-m)\delta}\right] - \sum_{m=1}^{n} d_m Q_{i,(n-m)\delta}. \tag{9}$$

For the purpose of simulations, only numerically significant coefficients are important.

Equation 6, for the z-transforms, can be rewritten in the form

$$C(z) = R \cdot Z[\{X_n\}]D(z),\ B(z) = R \cdot Z[\{Y_n\}]D(z), \tag{10}$$

equivalent to the convolution type relationships between the response factors X_n and Y_n and the conduction z-transfer function coefficients b_n, c_n, and d_n:

$$b_n = R\sum_{k=0}^{n} Y_{n-k} d_k,\ c_n = R\sum_{k=0}^{n} X_{n-k} d_k, \tag{11}$$

Equation 5 for the z-transforms $Z[\{Y_n\}]$ and $Z[\{X_n\}]$ now has the form:

$$\left.\frac{B(z)}{D(z)}\right|_{z=1} = \left.\frac{C(z)}{D(z)}\right|_{z=1} = 1, \tag{12}$$

which yields the following conditions for the dimensionless z-transfer function coefficients:

$$\sum_{n=0}^{\infty} b_n = \sum_{n=0}^{\infty} c_n = \sum_{n=0}^{\infty} d_n. \tag{13}$$

DETERMINING THE Z-TRANSFER FUNCTION COEFFICIENTS FROM THE RESPONSE FACTORS

On the basis of Equations 11 and 13, one may try to determine z-transfer function coefficients from the response factors Y_n and X_n. This is the most straightforward method; z-transfer functions obtained in this way are expected to "exactly" reproduce the output for any input function composed of straight-line segments, joining the points that represent its values at $t = n\delta$.

Assuming that z-transfer function coefficients with indices above some n are negligibly small, and $d_0 = 1$, we obtain the following set of linear equations:

$$b_0 + b_1 + b_2 + b_3 + \dots + b_n = 1 + d_1 + d_2 + d_3 + \dots + d_n \tag{14a}$$

$$b_0 = RY_0 \tag{14b}$$

$$b_1 = R(Y_1 + Y_0 d_1) \tag{14c}$$

$$b_2 = R(Y_2 + Y_1 d_1 + Y_0 d_2) \tag{14d}$$

$$b_3 = R(Y_3 + Y_2 d_1 + Y_1 d_2 + Y_0 d_3) \tag{14e}$$

....

$$b_n = R(Y_n + Y_{n-1}d_1 + Y_{n-2}d_2 + Y_{n-3}d_3 + ... + Y_0 d_n) \quad (14n)$$

$$0 = R(Y_{n+1} + Y_n d_1 + Y_{n-1}d_2 + Y_{n-2}d_3 + ... + Y_1 d_n) \quad (14n+1)$$

$$0 = R(Y_{n+2} + Y_{n+1}d_1 + Y_n d_2 + Y_{n-1}d_3 + ... + Y_2 d_n) \quad (14n+2)$$

....

$$c_0 + c_1 + c_2 + c_3 + ... + c_n = 1 + d_1 + d_2 + d_3 + ...d_n \quad (15a)$$

$$c_0 = RX_0 \quad (15b)$$

$$c_1 = R(X_1 + X_0 d_1) \quad (15c)$$

$$c_2 = R(X_2 + X_1 d_1 + X_0 d_2) \quad (15d)$$

$$c_3 = R(X_3 + X_2 d_1 + X_1 d_2 + X_0 d_3) \quad (15e)$$

....

$$c_n = R(X_n + X_{n-1}d_1 + X_{n-2}d_2 + X_{n-3}d_3 + ... + X_0 d_n) \quad (15n)$$

$$0 = R(X_{n+1} + X_n d_1 + X_{n-1}d_2 + X_{n-2}d_3 + ... + X_1 d_n) \quad (15n+1)$$

$$0 = R(X_{n+2} + X_{n+1}d_1 + X_n d_2 + X_{n-1}d_3 + ... + X_2 d_n) \quad (15n+2)$$

....

When the structure factors are calculated together with the resistance and the response factors, one may use conditions imposed by the structure factors on the z-transfer function coefficients as subsidiary equations (Kossecka 1998).

$$\sum_{n=1}^{\infty} nb_n - \sum_{n=1}^{\infty} nd_n = \frac{RC}{\delta}\varphi_{ie} \sum_{n=0}^{\infty} d_n \quad (16)$$

$$\sum_{n=1}^{\infty} nc_n - \sum_{n=1}^{\infty} nd_n = -\frac{RC}{\delta}\varphi_{ii} \sum_{n=0}^{\infty} d_n \quad (17)$$

Structure factors φ_{ii} and φ_{ie} are given by

$$\varphi_{ie} = \frac{1}{C}\int_V \rho c_p \theta(1-\theta)dv, \quad (18)$$

$$\varphi_{ii} = \frac{1}{C}\int_V \rho c_p (1-\theta)^2 dv, \quad (19)$$

where C is the total thermal capacity of the wall element of volume V,

$$C = \int_V \rho c_p dv, \quad (20)$$

and θ is the dimensionless temperature for the problem of steady-state heat transfer through the wall element, adiabatically cut from rest of the wall, for ambient temperatures $T_i = 0$ and $T_e = 1$. For plane walls, the products $C\varphi_{ii}$ and $C\varphi_{ie}$ are equivalent to the thermal mass factors introduced by Anderson (1985) (see also ISO 1991).

One may use more equations than the number of unknowns and apply minimizing procedures to get the solution. Maximum indices N_b, N_c, and N_d, of the coefficients b_n, c_n, and d_n, which should be included, depend on the specific dynamic thermal properties of a given wall assembly. In general, the total number of the numerically significant z-transfer function coefficients increases with the resistance and mass of the wall; however, it is not the rule. Trying different kinds of cut off, one should control the following quantity:

$$E_c = \frac{\sum_{n=0}^{N_c} c_n}{\sum_{n=0}^{N_d} d_n} - 1, \qquad E_b = \frac{\sum_{n=0}^{N_b} b_n}{\sum_{n=0}^{N_d} d_n} - 1 \quad (21)$$

$$E_r = E_c - E_b \quad (22)$$

E_c and E_b represent resultant errors of the z-transfer function coefficient calculations, and E_r represents the relative error of the heat flux in steady-state conditions, simulated using the z-transfer function method.

Solving Equation 11 for Y_n and X_n, with $d_0 = 1$, gives the recurrence formula, which may be used to additionally verify the solution obtained for the z-transfer function coefficients.

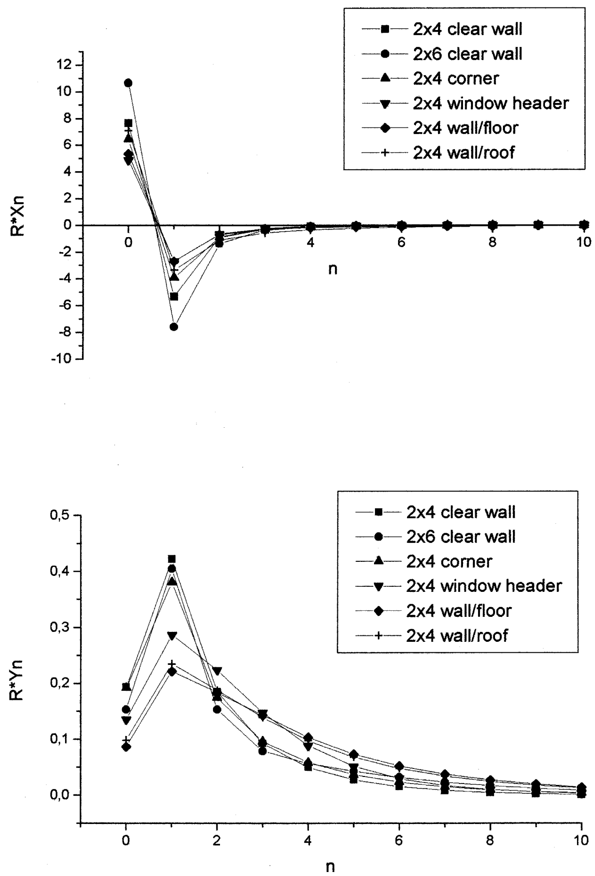
$$Y_0 = \frac{b_0}{R}, X_0 = \frac{c_0}{R} \quad (23)$$

$$Y_n = \frac{b_n}{R} - \sum_{k=1}^{n} Y_{n-k}d_k, X_n = \frac{c_n}{R} - \sum_{k=1}^{n} X_{n-k}d_k; n \geq 1 \quad (24)$$

Results of such a verification for the wood stud wall are presented in Appendix A.

CONDUCTION Z-TRANSFER FUNCTION COEFFICIENTS FOR COMMON WALL ASSEMBLIES

Dynamic thermal properties of 20 common wall assemblies were analyzed in the frames of the ASHRAE 1145-TRP project: “Modeling Two and Three-dimensional Heat Transfer through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs” (EE 2001). The list of wall assemblies considered includes clear walls and details of the wood- and steel-framed wall systems and also concrete and insulation structures: insulated concrete forms (ICF wall), sandwich walls with metal and plastic ties, and two-core block masonry walls, with or without insulation inserts. A represen-



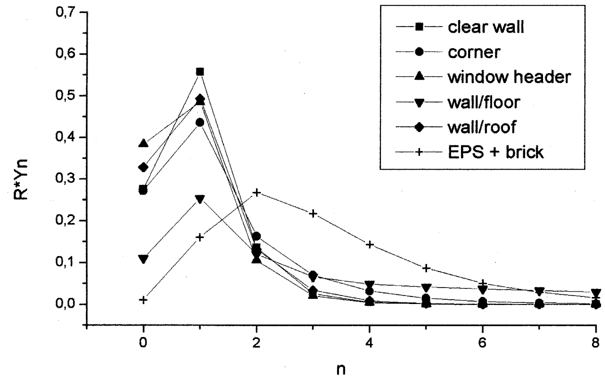


Figure 2 *Dimensionless response factors, $R{\cdot}Y_n$ for the 2×4 steel stud system wall assemblies.*

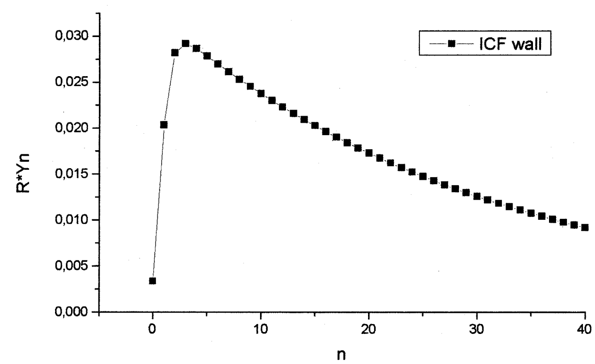


Figure 1 *Dimensionless response factors, $R{\cdot}X_n$ and $R{\cdot}Y_n$ for the wood-framed wall system assemblies.*

Figure 3 *Dimensionless response factors $R{\cdot}Y_n$ for the ICF wall.*

tative sample of details includes corners, intersections between above-grade wall, floor, and foundation, wall/roof interfaces, and the framing-in of walls around windows.

Drawings of all those wall assemblies, with simulation areas dimensioned, are included in the final report of the ASHRAE 1145-RP project (EE 2001). Drawings of most common structures may be found in Chapter 24, 1997 *ASHRAE Handbook—Fundamentals* (1997).

Response factors, resistances, and structure factors were calculated using the finite difference computer code HEATING 7.2 (Childs 1993) for boundary conditions of the first kind. Dimensionless, normalized response factors, given as $R{\cdot}X_n$ and $R{\cdot}Y_n$, for the 2×4 wood-stud system wall assemblies, are depicted in Figures 1 and 2, and $R{\cdot}Y_n$ for the ICF wall is shown in Figure 3. They represent relations of the responses to unit triangular temperature excitations after time $n\delta$, to the steady-state heat flux, due to the unit boundary temperature difference, equal to $1/R$.

The conduction z-transfer function coefficients were determined as the approximate solutions of the finite system of equations generated by Equations 14, 15, 16, and 17. The resultant error, E_r (Equation 21), was calculated when trying different types of the cut off to satisfy compatibility Equation 13 as well as possible. Modern calculation software used for this purpose allows one to easily examine different solutions of the problem, modifying the numbers of unknown variables. “Minimum error function,” used to find the solution of a system of N linear equations with M variables ($N \geq M$), shows very low sensitivity to the initial, input values.

The results are collected in Tables 1, 2, and 3. The maximum index of a coefficient does not exceed 5. The accuracy is within five decimal digits. The negative values of the coefficients b_n with higher indices, which appear for almost all lightweight wood- and steel-framed wall assemblies and empty concrete blocks, seem questionable at first sight. It was necessary to admit them to satisfy, with sufficient accuracy, compatibility Equation 13. For the steel-framed walls with heavy layers, brick or stucco, all b_n are positive. For the coefficients c_n and d_n, the sign sequence is always + and –, alternately. One should take into account, however, that negative values of some b_n does not mean that the impact of a temperature value may be “negative,” as temperatures enter into the expression for the current heat flux (Equation 9) not only through b_n and c_n but also through the preceding values of the heat flux itself.

TABLE 1
Wood Stud System—Clear Walls and Details, Z-Transfer Function Coefficients for Three-Dimensional Models

Wall assembly	R-value IP [SI]*	n	0	1	2	3	4
		b_n	0.19337	0.24476	-0.15501	0.01954	0.00097
2×4 clear wall	11.39127	c_n	7.64880	-12.33863	5.81307	-0.84437	0.02478
	[2.00486]	d_n	1.00000	-0.91447	0.23694	-0.01887	0.00004
		b_n	0.15322	0.25518	-0.21002	0.00873	
2×6 clear wall	17.48678	c_n	10.64983	-17.95135	8.12819	-0.60887	
	[3.07767]	d_n	1.00000	-0.97292	0.19791	-0.00718	
		b_n	0.19209	0.20133	-0.14086	0.00992	
2×4 corner	10.47675	c_n	6.43935	-9.92949	4.08246	-0.32742	
	[1.84391]	d_n	1.00000	-0.93234	0.20719	-0.01098	
		b_n	0.10141	0.16607	-0.01517	0.01302	
2×4 window	9.40385	c_n	5.05305	-7.80490	3.47950	-0.46234	
header	[1.65508]	d_n	1.00000	-0.99046	0.27744	-0.02166	
		b_n	0.08635	0.14395	-0.00347	-0.00869	
2×4 wall/floor	9.79471	c_n	5.34251	-7.49327	2.44246	-0.05226	
	[1.72387]	d_n	1.00000	-0.89680	0.13624		
		b_n	0.09812	0.14358	-0.01307		
2×4 wall/roof	9.40040	c_n	7.09777	-9.93976	3.20196		
	[1.65447]	d_n	1.00000	-0.92645	0.16872		

* ft²· °F·h/Btu [m²· K/W]

As an example, let's consider a 2×4 wood stud clear wall (see Table 1). Summing the z-transfer function coefficients gives

$$\sum_{n=0}^{4} b_n = 0.30364, \quad \sum_{n=0}^{4} c_n = 0.30365, \quad \sum_{n=0}^{4} d_n = 0.30365;$$
$$E_r = -0.00003.$$

The error E_r (see Equations 21 and 22) is here very small. (For most cases it's below 10^{-2}, for several cases below 10^{-5}; however, for two cases—a wood stud wall/roof and a steel stud wall/floor intersection—it reaches the value of 0.06). Results of the "reversibility test," using Equations (23) and (24), show very good compatibility of the response factors reproduced from the z-transfer function coefficients with those originally calculated for the three-dimensional model (see Appendix A, Table 1A). Heat flux test calculations (Figure 7) show excellent agreement of the results obtained using three-dimensional response factors and three-dimensional z-transfer function coefficients.

Some general conclusions concerning dynamic thermal properties represented by the response factors and the z-transfer function coefficients for subsequent types of walls are as follows:

1. **Wood-framed wall system.** Differences of the three-dimensional dimensionless z-transfer functions for clear walls and separated details are rather small; resistance differences are more significant.

2. **Steel-framed wall system.** Differences of the three-dimensional dimensionless z-transfer functions for clear walls and separated details are more significant here. The effect of additional layers of brick and stucco, together with the EPS foam layer, is substantial.

3. **ICF wall with three-dimensional internal concrete frame.** With a limited amount of concrete, it shows very specific dynamic thermal properties; Y_n response factors (and also X_n response factors with high indices) decay very slowly (see Figure 3) and b_n coefficients are very small. For simulations of the heat flow, to secure sufficient accuracy, one should use much more than 40 response factors; at the same time, maximum index for b_n, c_n, and d_n is, respectively, only 3, 4, and 2.

4. **Concrete/foam/concrete sandwich.** Resistance for the wall with plastic ties is significantly higher than for that with metal ties; dynamic properties are similar.

TABLE 2
Steel Stud System—Clear Walls and Details,
Z-Transfer Function Coefficients for Three-Dimensional Models

Wall assembly	R-value IP [SI]*)	*n*	0	1	2	3	4	5
2×4 steel stud, clear wall	8.79595 [1.54809]	b_n	0.27544	0.46780	-0.03740	-0.00348		
		c_n	5.38342	-5.43867	0.77712	-0.01951		
		d_n	1.00000	-0.32450	0.02704	-0.00018		
2×4 corner	5.28141 [0.92953]	b_n	0.27182	0.23733	-0.11860	0.00796	0.00019	
		c_n	3.32708	-4.25094	1.46144	-0.14221	0.00333	
		d_n	1.00000	-0.73120	0.13649	-0.00664	0.00004	
2×4 window, header	8.80252 [1.54924]	b_n	0.38340	0.40350	0.00274			
		c_n	2.00592	-1.25812	0.04204			
		d_n	1.00000	-0.21374	0.00358			
2×4 wall/floor	3.79581 [0.66806]	b_n	0.10985	0.12760	-0.14546	-0.01314		
		c_n	2.69068	-4.04151	1.51023	-0.07516		
		d_n	1.00000	1.14685	0.23109			
2×4 wall/roof	2.45400 [0.43190]	b_n	0.32878	0.33525	-0.08624	-0.00082		
		c_n	2.67848	-2.71613	0.65392	-0.03925		
		d_n	1.00000	-0.48144	0.05880	-0.00035		
2×4 steel stud +1-in EPS+brick	12.79160 [2.25132]	b_n	0.01014	0.15042	0.11515	0.00060		
		c_n	6.99557	-12.38192	7.07266	-1.50306	0.09593	
		d_n	1.00000	-0.96989	0.26922	-0.02015		
2×6 steel stud, clear wall	11.31363 [1.99120]	b_n	0.20008	0.48490	-0.02034	-0.00691		
		c_n	6.90969	-7.48203	1.26283	-0.03276		
		d_n	1.00000	-0.37815	0.03605	-0.00017		
2×6 steel stud + EPS + stucco	15.15703 [2.66764]	b_n	0.10508	0.34909	0.01933			
		c_n	8.12097	-11.41045	4.10657	-0.33939	0.00108	
		d_n	1.00000	-0.57984	0.05862			
2×6 steel stud + EPS + brick	15.46532 [2.72190]	b_n	0.00820	0.14244	0.12483	0.00222		
		c_n	8.28614	-14.88340	8.59382	-1.82738	0.10926	-0.00005
		d_n	1.00000	-0.97089	0.26764	-0.01836		

* Btu/h ft² °F [m² K/W]

TABLE 3
Concrete and Insulation Wall Assemblies Z-Transfer Function Coefficients for Three-Dimensional Models

Wall assembly	R-value IP [SI]*)	n	0	1	2	3	4
		b_n	0.00333	0.01647	0.00388	0.00022	
ICF – wall	11.23044	c_n	12.44518	-24.90756	14.40928	-1.92311	0.00012
	[1.97768]	d_n	1.00000	-1.16082	0.18473		
		b_n	0.02377	0.24051	0.14861	0.00778	
Sandwich wall	7.65912	c_n	35.22708	-49.09367	15.08613	-0.79797	
with metal ties	[1.34800]	d_n	1.00000	-0.70113	0.12272	-0.00002	
		b_n	0.01576	0.22377	0.16632	0.00908	
Sandwich wall	10.58216	c_n	50.74143	-71.97368	23.01010	-1.36291	
with plastic ties	[1.86246]	d_n	1.00000	-0.71157	0.12653	-0.00002	
		b_n	0.19225	0.15877	-0.15272	0.03684	
Empty	1.35549	c_n	5.98628	-9.65447	4.39335	-0.48275	
concrete blocks	[0.23857]	d_n	1.00000	-0.91110	0.15573	-0.00234	
		b_n	0.04707	0.13063	0.01402	0.03217	0.00389
Insulated	2.29137	c_n	9.72126	-16.99955	8.85120	-1.39633	0.05121
concrete blocks	[0.40328]	d_n	1.00000	-0.98904	0.23027	-0.01356	0.00012

* Btu/h ft^2 ºF [m^{2}K/W]

TABLE 4
Thermophysical Properties of the Three-Layer Equivalent Wall for the 2×4 Wood-Framed Wall

Layer N	R_n ft^2·°F·h/Btu [m^2·K/W]	C_n Btu/ft^2·°F [kJ/m^2·K]	l_n in. [m]	k_n Btu·in/h·ft^2·°F [W/m·K]	ρ_n lb/ft^3 [kg/m^3]	c_{pn} Btu/lb·°F kJ/kg·K
1	0.582 [0.102]	0.625 [12.771]	0.75 [0.019]	1.289 [0.186]	40 [640]	0.25 [1.048]
2	9.645 [1.698]	0.506 [10.345]	3.25 [0.083]	0.337 [0.049]	7.48 [120]	0.25 [1.048]
3	1.164 [0.205]	0.662 [13.528]	1 [0.025]	0.859 [0.124]	31.78 [508]	0.25 [1.048]

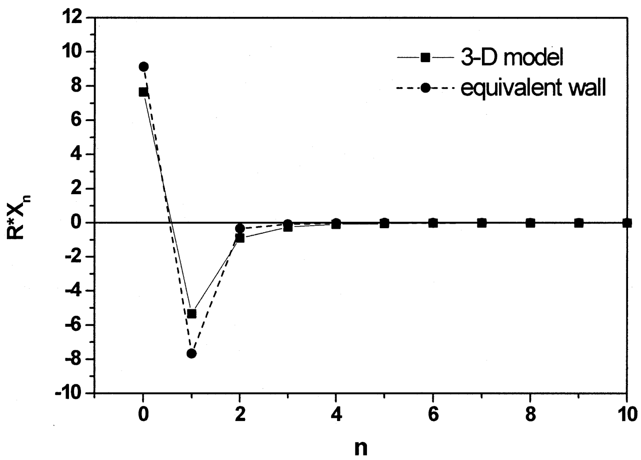


Figure 4 *Comparison of the dimensionless response factors $R \cdot X_n$ for the three-dimensional model and equivalent wall; 2×4 wood stud wall.*

5. **Heavy concrete blocks.** Dynamic thermal properties for empty blocks are essentially different compared with those for blocks filled with insulation.

COMPARISON WITH THE ONE-DIMENSIONAL EQUIVALENT WALL MODEL

Three-dimensional z-transfer function coefficients may be used effectively for the dynamic simulation of heating and cooling loads if the whole building simulation program enables this type of wall data input. An alternative way to perform whole building energy simulations, with professional programs such as DOE-2 or BLAST, is to employ a simple, one-dimensional model. In the equivalent wall method, developed by Kossecka and Kośny (1996, 1997), there is a multilayer wall of the same resistance, capacity, and structure factors as in the three-dimensional model of a wall assembly. An example of the three-layer equivalent wall, for the 2×4 wood stud wall, is presented in Table 4. Response factors for the three-dimensional model and for the equivalent wall are depicted in Figures 4 and 5; z-transfer function coefficients for the equivalent wall are listed in Table 5.

Although, in general, response factors for the equivalent wall look similar to those for the three-dimensional model, there are specific differences between them due to the fact that the effect of parallel heat flow paths of different conductance may not be completely reproduced by the simple, one-dimensional model. Uniform plane wall produces high internal response during the time of duration of the triangular temperature excitation, which is represented by comparatively high values of X_0 and X_1. At the same time, heat transfer through the plane wall is delayed compared with the wall with thermal bridges, which is represented by a comparatively low value of Y_0. Consequently, for the equivalent wall, values of the z-transfer function coefficients c_n with lowest indices are

TABLE 5
Dimensionless Z-Transfer Function Coefficients for the Equivalent Wall; 2×4 Wood-Framed Wall

n	b_n	c_n	d_n
0	0.05080	9.12250	1.00000
1	0.45323	-10.47223	-0.30686
2	0.19302	2.10285	0.00942
3	0.00547	-0.05063	-0.00002
4	0.00001	0.00005	

$\sum c_n = 0.70254$

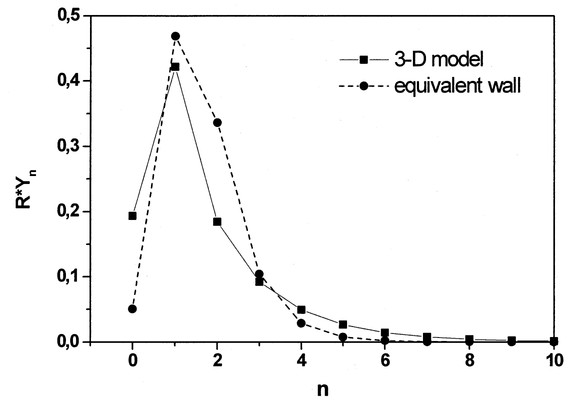


Figure 5 *Comparison of the dimensionless response factors $R \cdot Y_n$, for the three-dimensional model and equivalent wall; 2×4 wood stud clear wall.*

high, whereas values of the first coefficients b_n are low compared with the three-dimensional model.

The above-mentioned differences, however, do not greatly affect the values of the heat flux calculated for continuously varying temperature courses. Test simulations were performed using the sol-air temperature calculated for a vertical surface facing west on a sunny day in February in Warsaw as the outside surface temperature excitation (Figure 6). The temperature of the inside surface of a wall assembly represented periodic variations with the amplitude of 1.8°F (1°C), with a mean value of 68°F (20°C). The same daily temperature course was repeated several times to eliminate the effect of initial conditions.

The heat flux was simulated in three ways, using response factors for the three-dimensional model, z-transfer function coefficients for the three-dimensional model derived from the three-dimensional response factors, and z-transfer function coefficients for the equivalent wall. Results of the calculations for the 2×4 wood-framed system clear wall are presented in Figure 7. Steady-state (or massless wall) values are included to show dynamic effects—damping and the time lag.

Differences between the heat flux values calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients are almost invisible. An equivalent wall model produces some differences, though they are

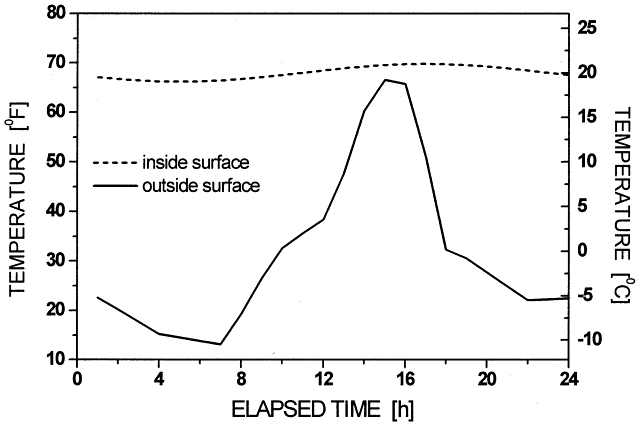


Figure 6 *Inside and outside surface temperature courses used for simulations.*

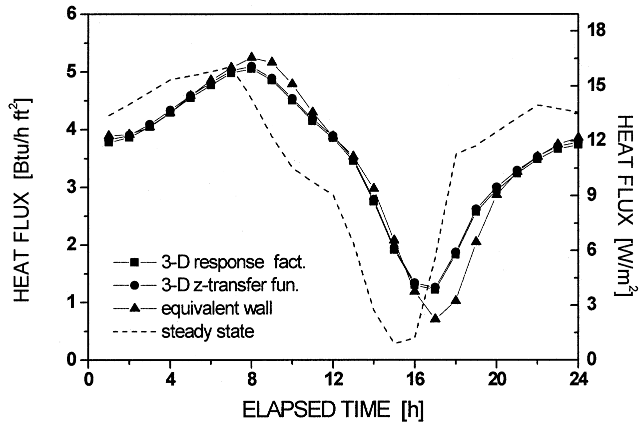


Figure 8 *Comparison of the heat flux simulation results using three methods for the corner; 2×4 wood stud wall.*

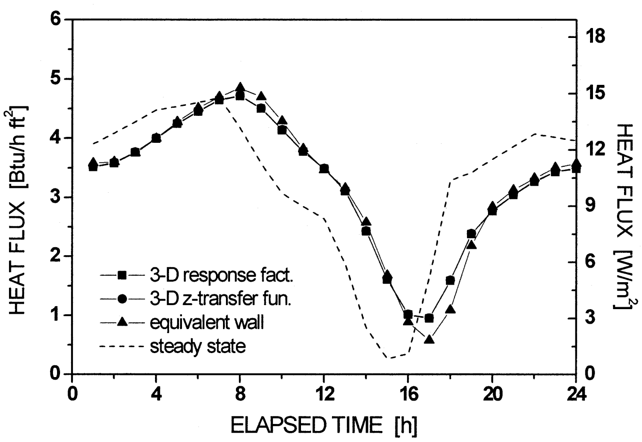


Figure 7 *Comparison of the heat flux simulation results using three methods for the 2×4 wood stud clear wall.*

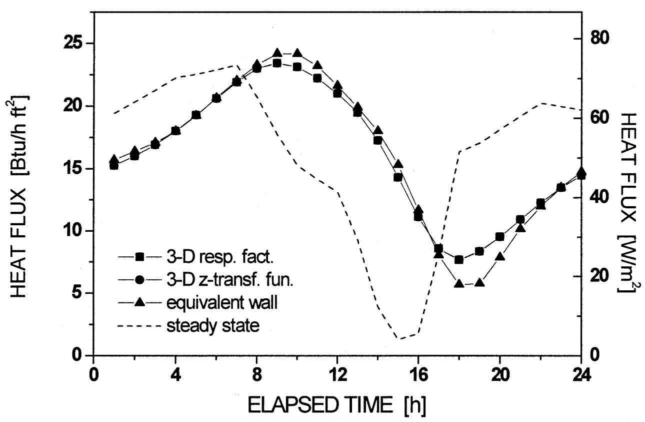


Figure 9 *Comparison of the heat flux simulation results using three methods for the insulated concrete blocks.*

small. An equivalent wall shows slightly "more dynamic" behavior. The same is to be observed for the corner (Figure 8). One should notice, however, that lightweight structures, such as the wood-framed walls, behave almost as "massless walls," so minor differences between different models are really unimportant here.

Figure 9 presents results of simulations for the massive concrete blocks filled with insulation, and Figure 10 shows the sandwich wall with metal ties. Compatibility of the three-dimensional model with the equivalent wall model is also very good here.

CONCLUSIONS

The method of derivation of the conduction z-transfer function coefficients from the response factors, for three-dimensional wall assemblies, gave satisfactory results.

The list of 20 wall assemblies that were analyzed includes clear walls and details of the wood- and steel-framed wall systems, insulated concrete forms (ICF wall), sandwich walls with metal and plastic ties, and two-core block masonry walls—with or without insulation inserts.

Response factors for three-dimensional models, calculated with a finite difference computer code for boundary conditions of the first kind, were used as the "input data" to determine z-transfer function coefficients from the primarily infinite set of linear equations, which includes relationships with the response factors and compatibility conditions. For each case, different kinds of the cut off were considered and minimization procedures were applied while seeking for the best solutions to satisfy compatibility conditions.

With an accuracy within five decimal digits, maximum index of a z-transfer function coefficient does not exceed 5. It was necessary to admit negative values of the coefficients b_n

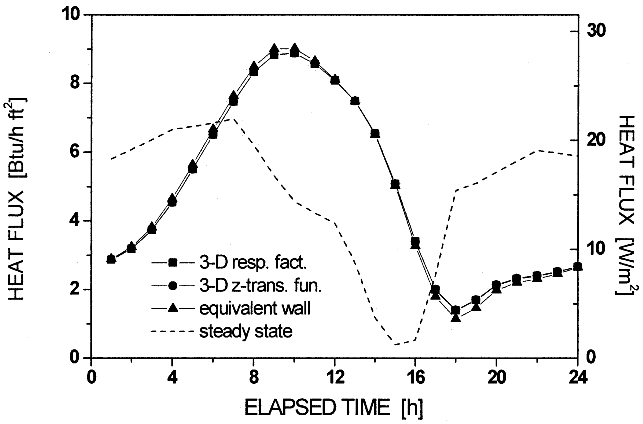


***Figure 10** Comparison of the heat flux simulation results using three methods for the sandwich wall with metal ties.*

with higher indices, for the lightweight wood- and steel-framed wall assemblies and for the empty concrete blocks, to satisfy with sufficient accuracy compatibility equations. For the coefficients c_n and d_n, the sign sequence is always + and – , alternately. The "reversibility test" shows very high accuracy in reproducing response factors from the z-transfer function coefficients.

A simple, one-dimensional model of the equivalent wall may give, in general, very good results in reproducing dynamic thermal properties of complex wall assemblies with thermal bridges. It does, however, produce some delay of the short-term heat flow transmitted through the wall and amplification of the internal response.

Test simulations performed with dynamic diurnal temperature excitations at the outside surface show perfect compatibility of the heat fluxes calculated using three-dimensional response factors and three-dimensional z-transfer function coefficients derived from the response factors and also good compatibility with those calculated using z-transfer function coefficients for the equivalent walls.

ACKNOWLEDGMENTS

This work was partly supported by ASHRAE research project 1145-TRP, "Modeling Two- and Three-Dimensional Heat Transfer Through Composite Wall and Roof Assemblies in Hourly Energy Simulation Programs."

NOMENCLATURE

δ = time instant [h]

$Q_{i,n\delta}$ = averaged over the interior surface heat flux at time $n\delta$; Btu/h·ft^2 [W/m^2]

$T_{i,n\delta}$ = interior temperature at time $n\delta$; °F [°C]

$T_{e,n\delta}$ = exterior temperature at time $n\delta$; °F [°C]

X_n, Y_n = response factors; Btu/h·ft^2·°F [W/m^2·K]

$Z[Q_i], Z[T_i], Z[T_e], Z[\{X_n\}], Z[\{Y_n\}], B(z), C(z), D(z)$ = z-transforms

b_n, c_n, d_n = dimensionless heat conduction z-transfer function coefficients

N_b, N_c, N_d = maximum index of numerically significant coefficients b_n, c_n, d_n respectively

E_b, E_c, $E_{r=}$ resultant errors of the z-transfer function calculations

R = thermal resistance per unit surface area of a wall or detail, ft^2·°F·h/Btu [m^2·K/W]

C = capacity per unit surface area of a wall or detail, Btu/ft^2·°F [kJ/m^2·K]

c_p = specific heat, Btu/lb·°F [J/m^3·K]

ρ = density, lb/ft^3 [kg/m^3]

V = volume of the wall element, ft^3 [m^3]

θ = dimensionless temperature

φ_{ii}, φ_{ie} = structure factors

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APPENDIX A

TABLE 1A
Comparison of the Original Dimensionless Three-Dimensional Response Factors with Those Reproduced from the Z-Transfer Function Coefficients—for a 2×4 Wood Stud Wall

n	Dimensionless response factors $R \cdot X_n$			Dimensionless response factors $R \cdot Y_n$		
	Reproduced From c_n, d_n	**$R \cdot X_n$ original values**	**$R \cdot \Delta X_n$**	**Reproduced from b_n, d_n**	**$R \cdot Y_n$ original values**	**$R \cdot \Delta Y_n$**
0	7.6488000	7.6488030	-0.0000027	0.1933700	0.1933737	-0.0000037
1	-5.3440320	-5.3440460	0.0000146	0.4215911	0.4215949	-0.0000039
2	-0.8861932	-0.8878742	0.0016811	0.1847053	0.1847107	-0.0000054
3	-0.2442194	-0.2442498	0.0000304	0.0922046	0.0922092	-0.0000046
4	-0.0897458	-0.0897582	0.0000124	0.0494752	0.0494763	-0.0000011
5	-0.0406983	-0.0406981	-0.0000002	0.0268640	0.0268632	0.0000008
6	-0.0205236	-0.0205199	-0.0000036	0.0145756	0.0145795	-0.0000038
7	-0.0108082	-0.0108069	-0.0000013	0.0078935	0.0078996	-0.0000061
8	-0.0057850	-0.0057861	0.0000010	0.0042696	0.0042756	-0.0000060
9	-0.0031149	-0.0031169	0.0000020	0.0023080	0.0023128	-0.0000048
10	-0.0016808	-0.0016829	0.0000020	0.0012473	0.0012508	-0.0000035
11	-0.0009077	-0.0009094	0.0000017	0.0006740	0.0006764	-0.0000024
12	-0.0004904	-0.0004916	0.0000012	0.0003642	0.0003657	-0.0000016
13	-0.0002649	-0.0002658	0.0000009	0.0001968	0.0001978	-0.0000010
14	-0.0001431	-0.0001437	0.0000006	0.0001063	0.0001069	-0.0000006
15	-0.0000773	-0.0000777	0.0000004	0.0000574	0.0000578	-0.0000004
16	-0.0000418	-0.0000420	0.0000002	0.0000310	0.0000313	-0.0000002
17	-0.0000226	-0.0000227	0.0000001	0.0000168	0.0000169	-0.0000001
18	-0.0000122	-0.0000123	0.0000001	0.0000091	0.0000091	-0.0000001
19	-0.0000066	-0.0000066	0.0000001	0.0000049	0.0000049	-0.0000001
20	-0.0000036	-0.0000036	0.0000000	0.0000026	0.0000027	-0.0000000
	Mean standard deviation σ_X = 0.000367			Mean standard deviation σ_Y = 0.0000032		